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# Transient effects in Mössbauer absorption caused by magnetic field reversal

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**Abstract.** The effect of magnetic field reversal on the transmission of Mössbauer radiation through a crystal as well as on the reaction yield is analysed. This reversal is shown to cause a strong flash of the transmitted radiation intensity followed by oscillations which attenuate over a time of the order of the nuclear lifetime. At the same time, there will be a suppression of inelastic channels and reactions for the nuclei lying near the face surface of the crystal. In contrast, for deep-lying nuclei there is an enhancement of the reaction yield due to a sharp increase of the recoilless radiation intensity in that region.

## 1. Introduction

A lot of papers (see the surveys [1–4]) are devoted to Mössbauer spectroscopy of soft ferromagnets subject to an external alternating magnetic field. When the crystal is placed in the radio-frequency (rf) magnetic field with frequency  $\Omega$ , the Mössbauer spectrum consists of a set of equidistant lines spaced by  $\Omega$ . Such a spectrum collapses to single line (doublet) at high frequencies. Pfeiffer [1–5] explained this effect by assuming that there are periodic reversals of the crystal magnetization governed by the rf field which ensure that there are reversals of the magnetic field at the nucleus between two values  $+h_0$  and  $-h_0$ . The corresponding step-wise-reversals model was constructed in [6–8]. The reversals of the magnetic field and forced vibrations were simultaneously taken into account in [9, 10]; that enabled us to explain the Pfeiffer results quantitatively [1, 5]. It was also shown [10, 11] that asymmetric reversals of the magnetic field at the nuclei lead to splitting of the absorption lines. New interesting effects arise in the case of low-frequency reversals, when the observed spectrum takes the form of a six-line Zeeman pattern with broadened lines [12, 13]. Transient beats of the absorption cross section are shown [8] to arise just after such reversals. The cross section even takes negative values within some time intervals, which implies the enhancement of the radiation intensity behind the absorber. The effect of the magnetic field reversal on the transmission of the Mössbauer radiation through a thick single crystal of iron borate,  $^{57}\text{FeBO}_3$ , has been studied recently by Shvyd'ko *et al* [12, 13]. This crystal is an antiferromagnet with slightly canted spins of opposite sublattices. Therefore its small magnetization is easily governed by an external magnetic field. The reversal of this magnetic field causes almost instant alteration of the directions of all atomic spins to the opposite ones and simultaneous reversal of the magnetic fields at the Mössbauer nuclei which are antiparallel to the spins. The reversals of the field at the nuclei between the two values  $\pm h_0$  were found in [12, 13] to be periodically repeated. The period  $T = 2 \mu\text{s}$  is

much greater than the lifetime  $\tau = 141$  ns of the  $^{57}\text{Fe}$  isotope. Therefore transient oscillations of the transmitted beam intensity, observed in [12, 13], practically attenuate between the neighbouring reversals, and reversals may be treated as isolated.

Moreover, Shvyd'ko *et al* [13] presented a theory, having, unfortunately, a number of disadvantages. While taking the incident wave as having frequency  $\omega$ , they used as the perturbation  $V(t)$  the interaction of the nucleus with the whole time-dependent electromagnetic wave packet arising in the crystal. It would be more correct to use a strict expression for  $V$  and consider the scattering of every plane component of the wave packets inside the scatterer, as is prescribed by collision theory (see, e.g., [14–16]). Something like that has been done by Kagan *et al* [17], calculating the response of the crystal to the pulse of synchrotron radiation. They used solutions of the dynamical scattering theory for every plane component of such pulses.

Additionally, Shvyd'ko *et al* [13] calculated the wave function for the whole system of the Mössbauer nuclei of the crystal only in first order of time-dependent perturbation theory. But when finding the wave function to describe the resonant scattering of  $\gamma$ -quanta by a crystal, we must take into account all of the powers of  $V(t)$  incorporated in the  $T$ -matrix. Furthermore, the nuclear wave function, used in [13], is a mixture of the wave functions of the ground and excited states of the nucleus. Calculating the mean value of the nuclear current-density operator  $\hat{j}_N$  with such wave functions, the authors of [13] write down only the transitional components of  $\hat{j}_N$  and arbitrarily omit the static nuclear moments both of the excited states and of the ground state, which are also contained in such a mean value.

Here we construct a strict dynamical theory for the transmission of Mössbauer radiation far from the Bragg condition through a crystal with reversing magnetization using a general collision theory in classical fields, constructed in [18]. It is based on the concept of the composite Hilbert space [19–21], which allows one to solve the time-dependent Schrödinger equation by the methods of stationary theory. Previously we used this approach for analysis of the scattering of  $\gamma$ -quanta and neutrons by crystals in alternating fields [22–25], neutron capture by nuclei in a laser beam [26], and x-ray acoustic [22, 27] and neutron acoustic [28] resonances.

A relatively new contribution here is the analysis of the influence of the magnetic field reversal on the yield of conversion electrons.

We should mention also the papers [29–32], which dealt with the decay of the nuclei excited by short pulses of synchrotron radiation. The duration of such pulses is much less than the nuclear lifetime  $\tau_N$ . Under these conditions, nuclear excitation and decay are independent processes. At the same time, in the case of Mössbauer scattering, the excitation and decay of the nuclei are intermixed and proceed within the same time interval,  $\sim \tau_N$ . Thus, the results from [29–32] cannot be applied here.

## 2. Wave functions

We shall consider a system which consists of the crystal, containing Mössbauer nuclei, plus the quantized electromagnetic field. Let the magnetic field at the Mössbauer nucleus reverse from  $+\mathbf{h}_0$  to  $-\mathbf{h}_0$  at the moment  $t = 0$ . In order to investigate transient phenomena caused by this reversal, we suppose that such reversals periodically repeat with period  $T$ , greatly exceeding the nuclear lifetime:

$$\mathbf{h}(t) = \mathbf{h}_0 f(t) = \mathbf{h}_0 f(t + T) \quad (1)$$

where

$$f(t) = \begin{cases} +1 & -T/2 < t < 0 \\ -1 & 0 < t < T/2. \end{cases} \quad (2)$$

Choosing the quantization axis  $\zeta$  along  $h_0$ , one can write down the interaction operator for the nucleus and the magnetic field as

$$\hat{V}_f(t) = -\gamma_\kappa \hat{I}_\zeta h_0 f(t) \quad (3)$$

where  $\hat{I}$  is the spin operator (in units of  $\hbar$ ) of the nucleus and  $\gamma_\kappa = g_\kappa \mu_N$  is the gyromagnetic ratio of the nucleus, which is a product of the nuclear magneton  $\mu_N$  and the  $g$ -factor  $g_\kappa$ ; the subscript  $\kappa = g$  for the ground nuclear state and  $\kappa = e$  for the excited one.

The Hamiltonian of the system will be

$$\begin{aligned} \hat{H} &= \hat{H}_0(t) + \hat{V} \\ \hat{H}_0(t) &= \hat{H}_N + \hat{H}_r + \hat{H}_e + \hat{H}_{ph} + \hat{V}_f(t) \end{aligned} \quad (4)$$

where  $\hat{H}_N$ ,  $\hat{H}_r$ ,  $\hat{H}_e$ , and  $\hat{H}_{ph}$  are respectively the Hamiltonians of the nucleus, quantized electromagnetic field, atomic electrons, and crystal lattice. The perturbation operator  $\hat{V}$  defines the interaction of the nucleus and atomic electrons with the quantized field, for which we use a standard quantum-mechanical expression (see, e.g., [33, 34]):

$$\begin{aligned} \hat{V} &= \hat{V}_r^N + \hat{V}_r^e \\ \hat{V}_r^{N(e)} &= -\frac{1}{c} \int \hat{j}_{N(e)}(\mathbf{r}) \cdot \hat{\mathbf{A}}(\mathbf{r}) d\mathbf{r} \end{aligned} \quad (5)$$

where  $\hat{j}_N(\mathbf{r})$  and  $\hat{j}_e(\mathbf{r})$  are the current-density operators of the nucleus and atomic electrons respectively;  $\hat{\mathbf{A}}(\mathbf{r})$  is the vector potential operator. Hereafter we use the Coulomb gauge for the quantized electromagnetic field.

The time-dependent Schrödinger equation with a periodic Hamiltonian  $\hat{H}(t) = \hat{H}(t+T)$  is easily solved in the composite Hilbert space of square integrable periodic functions  $\psi(q, t) = \psi(q, t+T)$  both of the spatial coordinates  $q$  and time  $t$ , where the scalar product of the functions  $\psi(q, t)$  and  $\varphi(q, t)$  is defined as [18–21]

$$\langle\langle \psi(q, t) | \varphi(q, t) \rangle\rangle = \frac{1}{T} \int_{-T/2}^{T/2} dt \int dq \psi^*(q, t) \varphi(q, t) \quad (6)$$

and the Floquet operators

$$\hat{\mathcal{H}}_0 = \hat{H}_0 - i\hbar \frac{\partial}{\partial t} \quad \hat{\mathcal{H}} = \hat{H} - i\hbar \frac{\partial}{\partial t} \quad (7)$$

are used instead of the Hamiltonians  $\hat{H}_0$  and  $\hat{H}$ .

The Mössbauer nucleus subject to the periodically reversing magnetic field is described by the Floquet wave function

$$\Psi_{I_\kappa M_\kappa}^N(t) = \psi_{I_\kappa M_\kappa}^N(t) e^{-i\varepsilon_{M_\kappa}^N t/\hbar} \quad (8)$$

where  $\psi(t) = \psi(t+T)$  is a periodic function of time. In the interval  $-T/2 < t < T/2$  it may be written as

$$\psi_{I_\kappa M_\kappa}^N(t) = |I_\kappa M_\kappa\rangle \exp\left(-\frac{i}{\hbar} \gamma_\kappa M_\kappa h_0 |t|\right) \quad (9)$$

where  $|I_\kappa M_\kappa\rangle$  stands for the stationary wave function of the nucleus in the  $\kappa$ th state with spin  $I_\kappa$  and its projection  $M_\kappa$  on the axis  $\zeta$ . The quasi-energies, which correspond to (9), coincide with

the energies of the nucleus in the absence of the magnetic field, i.e.  $\mathcal{E}_{M_g}^N = 0$  and  $\mathcal{E}_{M_c}^N = E'_0$ , where  $E'_0$  is the energy of the resonant level of the nucleus.

Let the incident  $\gamma$ -quantum have wave vector  $\mathbf{k}$ , polarization  $e$ , and frequency  $\omega = E/\hbar$ . Then the initial state of the whole system is described by the wave function

$$\begin{aligned} \Psi(q, t \rightarrow -\infty) &= \varphi_a(q, t) e^{-i\mathcal{E}_a t/\hbar} \\ \varphi_a(q, t) &= \phi_\alpha(t) |\mathbf{k}, e\rangle \quad \phi_\alpha(t) = \psi_{I_g M_g}^N(t) |\{v_s^0\}\rangle. \end{aligned} \quad (10)$$

Here the function  $\phi_\alpha(t)$  describes the initial state of the scatterer with the initial set of phonons  $\{v_s^0\}$ . The electron function is omitted for brevity. The corresponding quasi-energy  $\mathcal{E}_a$  is a sum of the photon energy  $E$  and the scatterer quasi-energy  $\epsilon_\alpha$ , which is the lattice vibrational energy.

At any moment, the wave function of the system is [18]

$$\Psi(q, t) = \psi_a^{(+)}(q, t) e^{-i\mathcal{E}_a t/\hbar} \quad (11)$$

where

$$\begin{aligned} \psi_a^{(+)}(q, t) &= \varphi_a(q, t) + \psi_{sc}^{(+)}(q, t) \\ \psi_{sc}^{(+)}(q, t) &= (\mathcal{E}_a + i\eta - \hat{\mathcal{H}}_0)^{-1} \hat{T} \varphi_a(q, t). \end{aligned}$$

Here  $\eta \rightarrow +0$ , and

$$\hat{T} = \hat{V} + \hat{V} \hat{G}(\mathcal{E}_a + i\eta) \hat{V} \quad (12)$$

is the transition operator with the Green operator

$$\hat{G}(z) = (z - \hat{\mathcal{H}})^{-1}. \quad (13)$$

First we shall consider the scattering of  $\gamma$ -quanta. In this case the final states of the system are described by the following eigenfunctions of the Floquet operator  $\hat{\mathcal{H}}_0$ :

$$|b, n\rangle = \varphi_b(t) e^{in\Omega t} \quad \varphi_b(t) = \phi_\beta(t) |\mathbf{k}', e'_{\lambda'}\rangle \quad \phi_\beta(t) = \psi_{I_g M_g}^N(t) |\{v'_s\}\rangle \quad (14)$$

with the corresponding quasi-energies

$$\mathcal{E}_{b,n} = \mathcal{E}_b + n\hbar\Omega \quad \mathcal{E}_b = E' + \epsilon_\beta \quad \epsilon_\beta = \sum_s \hbar\omega_s (v'_s + 1/2) \quad (15)$$

where  $E'$  is the energy of the scattered  $\gamma$ -quantum  $\mathbf{k}'$ ,  $e'_{\lambda'}$  and  $\epsilon_\beta$  is the vibrational energy of the lattice in the final state.

The radiation may be described by a wave function explicitly depending on  $\mathbf{r}$  and  $t$  with the aid of the electric field strength operator [14]

$$\hat{\mathbf{E}}(\mathbf{r}, t) = i \sum_{\mathbf{k}, \lambda} \sqrt{2\pi\hbar\omega_k} [\hat{a}_{\mathbf{k}, \lambda} e_{\lambda} e^{i\mathbf{k}\cdot\mathbf{r} - i\omega_k t} - \hat{a}_{\mathbf{k}, \lambda}^+ e_{\lambda}^* e^{-i\mathbf{k}\cdot\mathbf{r} + i\omega_k t}] \quad (16)$$

where  $\hat{a}_{\mathbf{k}, \lambda}$  and  $\hat{a}_{\mathbf{k}, \lambda}^+$  are annihilation and creation operators for the photon  $\mathbf{k}$ ,  $e_{\lambda}$ . In particular, the photon in the state  $|\mathbf{k}, e\rangle$  is described by

$$\mathbf{E}(\mathbf{r}, t) = \langle 0 | \hat{\mathbf{E}}(\mathbf{r}, t) | \mathbf{k}, e \rangle = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t} \quad (17)$$

where  $\mathbf{E}_0 = i\sqrt{2\pi\hbar\omega} e$ . By analogy, the radiation scattered into the  $\beta$ th channel is described by the wave function  $\Phi_{sc}^{(\beta)} = \langle \beta | \psi_{sc}^{(+)} \rangle$  or by

$$\mathbf{E}_{sc}(\mathbf{r}, t) = \langle 0 | \hat{\mathbf{E}}(\mathbf{r}, t) | \Phi_{sc}^{(\beta)} \rangle. \quad (18)$$

If the incident radiation is described by the wave packet

$$\mathbf{E}(z, t) = \mathbf{E}_0 \int_{-\infty}^{\infty} g(\omega) e^{i\mathbf{k}z - i\omega t} d\omega \quad (19)$$

then at any moment the whole system is described by the wave function (see also [14])

$$\int_{-\infty}^{\infty} g(\omega) \psi_a^{(+)}(q, t) d\omega \quad (20)$$

with  $\psi_a^{(+)}(q, t)$  given by equation (11).

In particular, when a  $\gamma$ -quantum is emitted by the nucleus excited at the initial moment  $t_0$ , we have

$$g(\omega) = -\frac{1}{2\pi i} \frac{e^{iE_0 t_0/\hbar}}{E - E_0 + i\Gamma_e/2} \quad (21)$$

where  $\Gamma_e$  and  $E_0$  are respectively the width and energy (including the possible Doppler shift) of the emitter level. Then such an emitted wave is described by

$$E(z, t) = E_0 \exp(-i\omega_0(t - t_0) - \Gamma_e(t - t_0 - z/c)/2\hbar) \theta(t - t_0 - z/c) \quad (22)$$

where

$$\theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0. \end{cases} \quad (23)$$

### 3. Scattering by one nucleus

The electromagnetic wave, coherently scattered by the  $j$ th nucleus, vibrating about the origin of the coordinate frame, in units of  $i(2\pi\hbar\omega)^{1/2}$ , is [8]

$$E_{sc}(\mathbf{r}, t)_{coh}^j = \sum_{\lambda'=\pm 1} \sum_{n=-\infty}^{\infty} f_{coh}^{(n)}(\mathbf{k}, e_\lambda; \mathbf{k}'_n, e'_{\lambda'})_j^N e'_{\lambda'} \frac{1}{r} e^{-i\omega_n t^*} \quad (24)$$

where  $t^* = t - r/c$  is the retarded time and

$$\omega' = \omega_n = \omega - n\Omega = k_n c \quad (25)$$

are possible frequencies of scattered photons. The coherent Raman scattering amplitude for scattering of  $\gamma$ -quanta by the nucleus with absorption ( $n < 0$ ) or emission ( $n > 0$ ) of  $n$  quanta of the alternating field with frequency  $\Omega$  is given by [8]

$$f_{coh}^{(n)}(\mathbf{k}, e_\lambda; \mathbf{k}', e'_{\lambda'})_j^N = -p_j \frac{e^{W_j(\mathbf{k}) - W_j(\mathbf{k}')}}{2I_g + 1} c^{-2} \sum_{M_g, M_e} \langle e | \hat{j}_{\lambda'}^N(\mathbf{k}') | g \rangle^* \langle e | \hat{j}_\lambda^N(\mathbf{k}) | g \rangle \\ \times \sum_{m=-\infty}^{\infty} \frac{a_{eg}^*(m - n) a_{eg}(m)}{E - E'_0 - m\hbar\Omega + i\Gamma/2} \quad (26)$$

where  $p_j$  is the probability of finding a resonant isotope at the  $j$ th site,  $\Gamma$  is the width of the resonant level,  $e^{-W}$  is the Lamb-Mössbauer factor,  $|\kappa\rangle = |I_\kappa, M_\kappa\rangle$  and

$$\hat{j}_\lambda^N(\mathbf{k}) = \int d\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} e_\lambda \cdot \hat{\mathbf{j}}_N(\mathbf{r}). \quad (27)$$

Also, we use the following Fourier coefficients:

$$a_{eg}(n) = \frac{1}{T} \int_{-T/2}^{T/2} dt e^{-int - \alpha_{eg}|t|} \quad (28)$$

where

$$\alpha_{eg} = (\gamma_g M_g - \gamma_e M_e) h_0 / \hbar. \quad (29)$$

Using the  $\zeta$ -function [33]

$$\zeta(x) = \int_0^{\infty} e^{-ix\xi} d\xi = \frac{1}{i} \frac{1}{x - i\eta} \quad (30)$$

we easily find the asymptotic formula for  $\alpha_{eg}$  as  $T \rightarrow \infty$ :

$$a_{eg}(n) = \frac{1}{iT} \left[ \frac{1}{n\Omega + \alpha_{eg} - i\eta} - \frac{1}{n\Omega - \alpha_{eg} + i\eta} \right]. \quad (31)$$

Substituting (26) into (24), we first take into account that

$$\sum_{n=-\infty}^{\infty} a_{eg}^*(m-n) e^{in\Omega t} = e^{i\alpha_{eg}|t|} e^{im\Omega t}. \quad (32)$$

Furthermore, as  $T \rightarrow \infty$  we replace the sum over  $m$  by the integral

$$\frac{1}{T} \sum_{m=-\infty}^{\infty} \rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tilde{\omega} \quad (33)$$

where  $m\Omega \rightarrow \tilde{\omega} = \omega' - \omega$  indicates the frequency alteration of the inelastically scattered photon. As a result, the coherently scattered wave becomes

$$\begin{aligned} \mathbf{E}_{sc}(\mathbf{r}, t)_{coh}^j &= -p_j \frac{e^{-W_j(\mathbf{k}) - W_j(\mathbf{k}')}}{2I_g + 1} \sum_{\lambda'=\pm 1} e'_{\lambda'} c^{-2} \sum_{M_e, M_g} \langle e | \hat{j}_{\lambda'}^N(\mathbf{k}') | g \rangle^* \langle e | \hat{j}_{\lambda}^N(\mathbf{k}) | g \rangle e^{i\alpha_{eg}|t^*|} \\ &\times \left( \frac{1}{2\pi i} \right) \int_{-\infty}^{\infty} d\tilde{\omega} \frac{e^{i\tilde{\omega}t^*}}{E - E'_0 - \hbar\tilde{\omega} + i\Gamma/2} \\ &\times \left( \frac{1}{\tilde{\omega} + \alpha_{eg} - i\eta} - \frac{1}{\tilde{\omega} - \alpha_{eg} + i\eta} \right) \frac{e^{-i\omega t^*}}{r}. \end{aligned} \quad (34)$$

We shall consider only the case in which  $\hbar|\alpha_{eg}| \gg \Gamma$ , i.e. where the Zeeman spectrum has a well-resolved hyperfine structure. Also, we assume the following resonant condition for incident  $\gamma$ -quanta to be fulfilled:

$$E \simeq E'_0 + \hbar\alpha_{eg}. \quad (35)$$

Then only the isolated transition  $M_g \rightarrow M_e$  is excited in the nucleus placed in the constant magnetic field  $+\mathbf{h}_0$ . In this case all of the other nuclei with  $M'_g \neq M_g$  do not ‘feel’ the Mössbauer radiation at all. In contrast, after the reversal of the field from  $+\mathbf{h}_0$  to  $-\mathbf{h}_0$ , only the transition  $-M_g \rightarrow -M_e$  becomes resonant.

By definition, the coherent scattered wave (34) is

$$\mathbf{E}_{sc}(\mathbf{r}, t)_{coh}^j = \frac{1}{2I_g + 1} \sum_{M_g} \mathbf{E}_{sc}(\mathbf{r}, t)_{M_g \rightarrow M_g}^j \quad (36)$$

where  $\mathbf{E}_{sc}(\mathbf{r}, t)_{M_g \rightarrow M_g}^j$  is the wave scattered by the  $j$ th nucleus, having the same quantum number  $M_g$  in both the initial and final states. It is instructive to extract the contributions  $\mathbf{E}_{sc}(\mathbf{r}, t)_{M_g \rightarrow M_g}^j$  to (36) from the nuclei with quantum numbers  $+M_g$  or  $-M_g$ .

Performing a contour integration in (34) for nuclei with quantum number  $+M_g$ , one gets

$$\mathbf{E}_{sc}(\mathbf{r}, t)_{M_g \rightarrow M_g}^j = \sum_{\lambda'} e'_{\lambda'} f_{gg}(\mathbf{k}, e_{\lambda}; \mathbf{k}', e'_{\lambda'})_j^N \frac{1}{r} \left\{ (1 - \theta(t^*)) e^{-i\omega t^*} + \theta(t^*) e^{-i\omega_0^- t^* - \Gamma t^*/2\hbar} \right\} \quad (37)$$

where  $f_{gg}$  is the elastic scattering amplitude of  $\gamma$ -quanta by the nucleus  $+M_g$  situated in the constant field  $\mathbf{h}_0$ :

$$f_{gg}(\omega) = f_{gg}(\mathbf{k}, e_\lambda; \mathbf{k}', e'_\lambda)_j^N = -p_j \frac{e^{-W_j(\mathbf{k}') - W_j(\mathbf{k})}}{2I_g + 1} \frac{c^{-2} \langle e | \hat{j}_\lambda^N(\mathbf{k}') | g \rangle^* \langle e | \hat{j}_\lambda^N(\mathbf{k}') | g \rangle}{E - E_0^+ + i\Gamma/2} \quad (38)$$

and the  $\omega_0^\pm = E_0^\pm / \hbar$  stand for the resonant frequencies of the transitions  $\pm M_g \rightarrow \pm M_e$  in this field with transition energies  $E_0^\pm = E'_0 \pm \hbar\alpha_{eg}$ .

Thus, when  $t < 0$ , the nuclei, having initial quantum number  $M_g$ , do not 'anticipate' a future reversal of the magnetic field in accordance with the causality principle. They scatter as if in the stationary state. After the reversal ( $t > 0$ ), only previously excited nuclei of this sort continue to decay, yielding the exponentially attenuating tail of the scattered wave. It has the carrier frequency  $\omega_0^-$ , which already corresponds to the de-excitation transition  $M_e \rightarrow M_g$  in the new field  $-\mathbf{h}_0$ .

For the nuclei in the initial state  $|-M_g\rangle$ , the scattered wave is

$$\mathbf{E}_{sc}(\mathbf{r}, t)_{-M_g \rightarrow -M_g}^j = \sum_{\lambda'} e'_{\lambda'} f_{-g, -g}(\mathbf{k}, e_\lambda; \mathbf{k}', e'_{\lambda'})_j^N \frac{1}{r} \left\{ e^{-i\omega t^*} - e^{-i\omega_0^+ t - \Gamma t^*/2\hbar} \right\} \theta(t^*) \quad (39)$$

where  $f_{-g, -g}(\omega)$  is defined by equation (38) with  $e, g \rightarrow -e, -g$ . This wave arises only after the reversal. It contains the same ordinary wave  $\sim e^{-i\omega t^*}$  as in the stationary case and an additional transient attenuating wave with carrier frequency  $\omega_0^+$  associated with the transition  $-M_g \rightarrow -M_e$  in the negative field  $-\mathbf{h}_0$ .

Later we shall analyse the transmission of the radiation through the slab when  $\mathbf{k} = \mathbf{k}'$  is directed perpendicularly to  $\mathbf{h}_0$ . Then only  $\gamma$ -quanta having linear polarization  $e$  perpendicular to  $\mathbf{h}_0$  provide transitions with  $M_e - M_g = 0$  in  $^{57}\text{Fe}$  and those with  $e$  parallel to  $\mathbf{h}_0$  take part in transitions  $M_e - M_g = \pm 1$ . Such eigen-polarization does not change when  $\gamma$ -quanta pass through the crystal. If  $\mathbf{k} = \mathbf{k}'$  and  $e_\lambda = e'_\lambda = e$ , then the amplitudes  $f_{gg} = f_{-g, -g} = 2f_{coh}$ .

Substitution of (37) and (39) into (36) gives the complete coherent wave scattered to the forward direction:

$$\mathbf{E}_{sc}(\mathbf{r}, t)_{coh}^j = E_0 f_{coh}(\mathbf{k}, e; \mathbf{k}, e)_j^N \frac{1}{r} \left\{ e^{-i\omega t^*} + \left( e^{-i\omega_0^- t^*} - e^{-i\omega_0^+ t} \right) e^{-\Gamma t^*/2\hbar} \theta(t^*) \right\}. \quad (40)$$

It is interesting that the transient wave packets are always concentrated at the resonant frequencies  $\omega_0^\pm$ , independently of the frequency  $\omega$  of the incident photons.

The result (39) may be also obtained by means of the stationary scattering theory, by taking the incident wave

$$\mathbf{E}(z, t) = E_0 e^{-i\omega(t-z/c)} \theta(t - z/c). \quad (41)$$

It interacts with the nucleus at the origin of the coordinate frame only after the reversal. Therefore all of the scattering proceeds as if the field  $-\mathbf{h}_0$  exists all the time,  $-\infty < t < \infty$ . The wave (41) may be rewritten as

$$\mathbf{E}(z, t) = E_0 \int_{-\infty}^{\infty} g(\omega') e^{-i\omega'(t-z/c)} d\omega' \quad (42)$$

with

$$g(\omega') = -\frac{1}{2\pi i} \frac{1}{\omega' - \omega + i\eta}. \quad (43)$$

The wave scattered by the  $-M_g$ -nucleus is given by

$$\mathbf{E}_{sc}(\mathbf{r}, t)_{-M_g \rightarrow -M_g}^j \sim \int_{-\infty}^{\infty} g(\omega') f_{-g, -g}(\omega') e^{-i\omega' t^*} d\omega'. \quad (44)$$



Performing the contour integration we return to expression (39).

The wave (40) is generated by the incident plane wave with frequency  $\omega$ . If the target is interacting with the wave packet (22), the scattered one takes the form

$$\vec{\mathcal{E}}_{sc}(\mathbf{r}, t)_{coh}^j \sim \frac{1}{(\omega_0 - \omega_0^+)} e^{-\Gamma(t^* - t_0)/2\hbar} \left\{ \left( e^{-i\omega_0(t^* - t_0)} - e^{-i\omega_0^+(t^* - t_0)} \right) \theta(t^* - t_0) + \left( e^{-i\omega_0^- t^*} - e^{-i\omega_0^+ t} \right) \left( e^{i\omega_0 t_0} - e^{i\omega_0^+ t} \right) \theta(-t_0) \theta(t^*) \right\}. \quad (45)$$

Here we see the ordinary wave,  $\sim\theta(t^* - t_0)$ , which exists also in the stationary case, and additional transient ones,  $\sim\theta(-t_0)$ , which arise only if the incident wave packet reaches the scatterer prior to the field reversal. Every part of  $\vec{\mathcal{E}}_{sc}$  has the same attenuation. Temporary beats of the scattered intensity depend strongly on the initial moment  $t_0$ .

#### 4. Transmission through the crystal

We shall analyse the transmission of Mössbauer radiation through a crystal slab far from the Bragg condition, when any diffraction is unimportant. Let the crystal occupy the space  $0 \leq z \leq \mathcal{D}$  and consist of  $N$  infinite layers of elementary cells, which are perpendicular to the axis  $z$ . The lattice vector  $\mathbf{l} = l_1 \mathbf{a}_1 + l_2 \mathbf{a}_2 + l_3 \mathbf{a}_3$ , where  $\mathbf{a}_i$  are the basis vectors;  $l_1, l_2 = 0, \pm 1, \pm 2, \dots, \pm \infty$  and  $0 \leq m = l_3 \leq N - 1$ . The basis vectors  $\mathbf{a}_1$  and  $\mathbf{a}_2$  lie in the plane  $x, y$ . The thickness of one layer of elementary cells  $d = \mathbf{a}_3 \cdot \mathbf{e}_z$ , where  $\mathbf{e}_z$  is the unit vector along the axis  $z$ . The crystal thickness is  $\mathcal{D} = Nd$ . Let incident  $\gamma$ -quanta have the wave vector  $\mathbf{k} = \{0, 0, k\}$  and eigen-polarization  $e$ .

Interference of all of the spherical waves scattered by the atoms of the face layer of elementary cells ( $m = 0$ ) gives a number of scattered plane waves. Far from the Bragg condition, we are interested only in the wave scattered to the forward direction:

$$\mathbf{E}_{sc}^{(0)}(z, t) = i \left\{ \mathcal{F} e^{-i\omega \tilde{t}} + \mathcal{F}^N \left( e^{-i\omega_0^- \tilde{t}} - e^{-i\omega_0^+ \tilde{t}} \right) e^{-\Gamma \tilde{t}/2\hbar} \theta(\tilde{t}) \right\} \mathbf{E}_0 \quad (46)$$

where  $\tilde{t} = t - z/c$  and  $\mathcal{F}$  is the dimensionless straightforward scattering amplitude of the incident wave  $\mathbf{E}^{(0)}(z, t) = \mathbf{E}_0 \exp(-i\omega \tilde{t})$  with  $\mathbf{E}_0 \sim e$ , scattered by a single layer. It equals the sum of the Rayleigh and nuclear scattering amplitudes:

$$\mathcal{F} = \mathcal{F}^R + \mathcal{F}^N \quad (47)$$

which are determined by the formula

$$\mathcal{F}^{R(N)} = \frac{2\pi}{k_z(v_0/d)} \sum_j f_{coh}(\mathbf{k}, \mathbf{e}; \mathbf{k}, \mathbf{e})_j^{R(N)} \quad (48)$$

where  $v_0$  is the volume of the elementary cell. The summation in (48) is carried out over all atoms within the elementary cell. The nuclear amplitude may be written as

$$\mathcal{F}^N = -\frac{b_{eg}/N}{E - E_0^+ + i\Gamma/2} \quad (49)$$

where  $b_{eg}$  is the thickness parameter of the absorber for the isolated transition  $M_g \rightarrow M_e$ :

$$b_{eg} = \frac{\sigma_0 \Gamma}{4} e^{-2W_a} J_{eg}(\pi/2) n_0. \quad (50)$$

$\sigma_0$  stands for the resonant cross section,  $J_{eg}(\pi/2)$  determines the relative intensity of the line  $M_e \rightarrow M_g$ , and  $n_0$  is the number of Mössbauer isotopes per unit square of the absorber. The wave incident on the next layer,  $\mathbf{E}^{(1)}(\tilde{t})$ , is a sum of the incident wave  $\mathbf{E}^{(0)}(\tilde{t})$  and the scattered wave  $\mathbf{E}_{sc}^{(0)}(\tilde{t})$ . This wave will be scattered by the next layer, etc. Repeating such a

procedure, we can find the electromagnetic wave at any point of the crystal. It consists of the ordinary wave  $\sim \exp(i\mathbf{K} \cdot \mathbf{r})$  with the wave vector  $\mathbf{K} = \{0, 0, k + \mathcal{F}/d\}$  and some transient wave packets, which appear at  $t > 0$  and ‘feel’ only the constant field  $-\mathbf{h}_0$ . Therefore its scattering proceeds as in the stationary case with the field  $-\mathbf{h}_0$ , existing throughout the whole time period without any reversals. In other words, the transient waves  $\sim e^{i\omega_0^\pm \tilde{t}}$  do not give rise to other transient ones when colliding with the nuclei of the crystal. They are produced only by the ordinary wave  $\exp(i\mathbf{K} \cdot \mathbf{r} - i\omega t)$ . Then the transient part of the wave scattered by the  $m'$ th layer will be

$$\mathbf{E}_{sc}^{(m')}(\tilde{t})_{tr} = i\mathcal{F}^N \left\{ e^{ik_0^-(z-m'd)-i\omega_0^-t} - e^{ik_0^+(z-m'd)-i\omega_0^+t} \right\} e^{-\Gamma\tilde{t}/2h}\theta(\tilde{t})\mathbf{E}_0 e^{iK m'd} \tag{51}$$

where  $k_0^\pm = \omega_0^\pm/c$ .

In order to analyse the subsequent transmission of this wave packet through the crystal, we expand it in terms of the plane waves:

$$e^{-i\omega_0^\pm \tilde{t} - \Gamma\tilde{t}/2h}\theta(\tilde{t}) = -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} dE' \frac{e^{-i\omega'\tilde{t}}}{E' - E_0^\pm + i\Gamma/2}. \tag{52}$$

Note again that this wave packet ‘feels’ only the field  $-\mathbf{h}_0$ , as if it were constant throughout the whole time period  $-\infty < t < \infty$ . Hence, transmission of every plane component of (52) is described by the stationary dynamical theory. That is, the wave  $\sim e^{-i\omega'\tilde{t}}$  generates in the region  $z > m'd$  a plane wave  $\sim e^{iK'z - i\omega't}$  with  $K' = k' + \mathcal{F}'/d$  ( $\mathcal{F}'$  is given by (47)–(49) with  $E$  replaced by  $E'$ ). Then the transient wave (51), reaching the layer  $m > m'$ , transforms to the wave

$$\mathbf{E}_{sc}^{(m')}(\mathbf{z})_{tr} \rightarrow i\mathcal{F}^N (I_-(y_{m'}) - I_+(y_{m'})) \exp\left[\frac{-ib_{eg}(m')}{E - E_0^+ + i\Gamma/2}\right] e^{im\mathcal{F}^R} \mathbf{E}_0 \tag{53}$$

where  $y_{m'} = 1 - m'/m$  and the  $I_\pm$  designate the integrals

$$I_\pm(y_{m'}) = -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} dE' \frac{e^{-i\omega'\tilde{t}}}{E' - E_0^\pm + i\Gamma/2} \exp\left[-\frac{ib_{eg}(m)y_{m'}}{E' - E_0^\pm + i\Gamma/2}\right]. \tag{54}$$

$b_{eg}(m) = (m/N)b_{eg}$  is the thickness parameter for  $m$  layers. The wave incident on the  $m$ th layer of the crystal  $\mathbf{E}^{(m)}(\mathbf{z}, t)$  will be the sum of the ordinary wave and all partial transient waves emerging at the layers with numbers  $m' = 0, 1, \dots, m - 1$ . One can replace the sum over  $m'$  by the integral

$$\frac{1}{m} \sum_{m'=0}^{m-1} \rightarrow \int_0^1 dy \quad y_{m'} \rightarrow y. \tag{55}$$

Then for  $\mathbf{E}^{(m)}(\mathbf{z}, t)$  with  $z \approx md$ , one gets the expression

$$\mathbf{E}^{(m)}(\mathbf{z}, t) = \mathbf{E}_{ord}(\mathbf{z}, t) + \mathbf{E}_{tr}(\mathbf{z}, t)^{(+)} + \mathbf{E}_{tr}(\mathbf{z}, t)^{(-)} \tag{56}$$

where

$$\mathbf{E}_{ord}(\mathbf{z}, t) = \exp\left(-\frac{ib_{eg}(m)}{E - E_0^+ + i\Gamma/2}\right) e^{-i\omega\tilde{t}} e^{im\mathcal{F}^R} \mathbf{E}_0 \tag{57}$$

is the ordinary wave and the waves

$$\mathbf{E}_{tr}(\mathbf{z}, t)^{(\pm)} = \pm \frac{ib_{eg}(m)}{E - E_0^+ + i\Gamma/2} \int_0^1 dy \exp\left(-\frac{ib_{eg}(m)(1-y)}{E - E_0^+ + i\Gamma/2}\right) I_\pm(y) e^{im\mathcal{F}^R} \mathbf{E}_0 \tag{58}$$

are the transient waves generated by the nuclei that are in the initial states  $\mp M_g$ .

Following [35], we calculate the integrals  $I_{\pm}$  with the aid of the familiar relation

$$\exp\left[\frac{u}{2}\left(v - \frac{1}{v}\right)\right] = \sum_{k=-\infty}^{\infty} J_k(u)v^k \quad (59)$$

where  $J_k(u)$  is the Bessel function of the  $k$ th order. Putting

$$z = E' - E_0^+ + i\frac{\Gamma}{2} \quad u = 2\sqrt{b_{\text{eg}}(m)y\tilde{t}/\hbar} \quad v = -i\sqrt{\frac{\tilde{t}/\hbar}{b_{\text{eg}}(m)y}}z \quad (60)$$

and performing contour integration on the complex  $z$ -plane one gets

$$I_{\pm}(y) = J_0\left(2\sqrt{b_{\text{eg}}(m)y\tilde{t}/\hbar}\right)e^{-i\omega_0^{\pm}\tilde{t} - \Gamma\tilde{t}/2\hbar}\theta(\tilde{t}). \quad (61)$$

Equations (56)–(61) determine the coherent electromagnetic wave inside the crystal at  $z \simeq md$ . When  $m = N$  and  $z > \mathcal{D}$ , they describe the transmitted wave  $\mathbf{E}^{(N)}(z, t)$ , excited by the incident wave with frequency  $\omega$ . Correspondingly, the incident wave packet (19) generates the transmitted wave

$$\vec{\mathcal{E}}_{tr}(z, t) = \int_{-\infty}^{\infty} g(\omega)\mathbf{E}^{(N)}(z, t) d\omega. \quad (62)$$

The flux density of  $\gamma$ -quanta  $\sim |\vec{\mathcal{E}}|^2$  still depends on  $t_0$ . Averaging it over  $t_0$ , we take into account that

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} g(\omega)g(\omega')^* dt_0 = |g(\omega)|^2\delta(\omega - \omega') \quad (63)$$

with  $|g(\omega)|^2$  proportional to the energy distribution of the incident recoilless radiation

$$w_e^{(0)}(E) = A \frac{\Gamma_e/2\pi}{(E - E_0)^2 + (\Gamma_e/2)^2}. \quad (64)$$

The amplitude  $A$  determines the relative intensity of the phononless line in the incident beam. In [13] the incident radiation has been filtered through a polarizer completely absorbing  $\gamma$ -quanta with one polarization. For such a case,  $A = 0.5e^{-2W_e}$ , where  $e^{-2W_e}$  is the Debye–Waller factor of the emitter. For the Co(Cr) source of  $\gamma$ -quanta used in the experiment in [13],  $e^{-2W_e} = 0.75$  at  $T = 300$  K [36]. Therefore, in the numerical calculations we took  $A = 0.37$ .

It is useful to introduce the dimensionless parameters

$$\begin{aligned} \beta_{\text{eg}} &= 4b_{\text{eg}}/\Gamma & \tau &= \Gamma\tilde{t}/\hbar & \gamma_e &= \Gamma_e/\Gamma & a_{\text{eg}} &= 2\hbar\alpha_{\text{eg}}/\Gamma \\ x_0 &= 2(E_0 - E_0^+)/\Gamma & x &= 2(E - E_0^+)/\Gamma \end{aligned} \quad (65)$$

where  $\tau$  is the time in units of the nuclear lifetime  $\hbar/\Gamma$  and  $x_0$  is the detuning parameter. Then the averaged flux density of all transmitted  $\gamma$ -quanta in units of  $j_0e^{-\mu_e\mathcal{D}}$  is

$$j(t) = B + A \int_{-\infty}^{\infty} \frac{(\gamma_e/\pi) dx}{(x - x_0)^2 + \gamma_e^2} \exp\left(-\frac{\beta_{\text{eg}}}{x^2 + 1}\right) |1 + F(x, \tau)|^2 \quad (66)$$

where  $B = 1 - A$  stands for the background,  $j_0$  is the flux density of incident  $\gamma$ -quanta, and  $\mu_e = 2 \text{Im } \mathcal{F}^R/d$  is the photoelectron absorption coefficient. The function  $F(x, \tau)$  describes transient beats following the reversal of the magnetic field:

$$F(x, \tau) = \sin(a_{\text{eg}}\tau/2) \left(\frac{\beta_{\text{eg}}}{x + i}\right) e^{i(x+a_{\text{eg}})\tau/2 - \tau/2} \int_0^1 dy J_0\left(\sqrt{\beta_{\text{eg}}\tau}y\right) \exp\left(\frac{i\beta_{\text{eg}}y/2}{x + i}\right) \theta(\tau). \quad (67)$$

If  $\tau < 0$ , the function  $F(x, \tau) = 0$  and equation (66) coincides with the familiar integral for the transmitted beam intensity (see, e.g., [37]). In the case of thin crystals ( $\beta \ll 1$ ), expanding (66), (67) in terms of  $\beta$  and retaining only linear terms, one has

$$j(t) = 1 - n_0 \bar{\sigma}_a(\xi, \tau) + O(\beta^2) \tag{68}$$

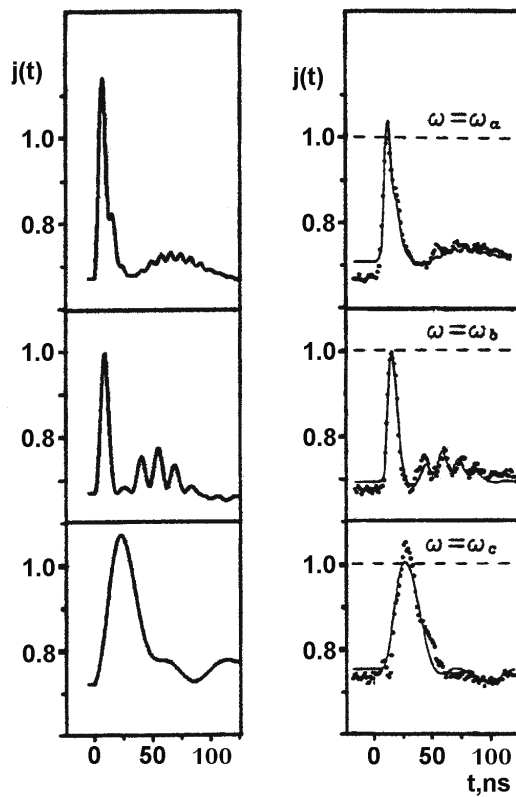
where  $n_0$  is the number of resonant isotopes per unit square and  $\bar{\sigma}_a(\xi, \tau)$  is the instantaneous absorption cross section of Mössbauer radiation averaged over the energy distribution (64). For  $\gamma_e = 1$ ,

$$\bar{\sigma}_a(\xi, \tau) = \left( \frac{\beta_{eg}}{2n_0} \right) \frac{A}{1 + (\xi - \xi_{eg})^2} \{ 1 - 2 \sin(\xi_{eg} \tau) [(\xi - \xi_{eg}) \cos(\xi \tau) + \sin(\xi \tau)] e^{-\tau} \theta(\tau) \} \tag{69}$$

where

$$\xi = (x_0 + a_{eg})/2 \quad \xi_{eg} = a_{eg}/2.$$

Formula (69) agrees with our previous more exact calculations [8] of the absorption cross section, in which all of the transitions (including nonresonant ones) were taken into account.



**Figure 1.** The transmitted beam intensity versus time after the reversal of the magnetic field for the transitions *a*:  $-1/2 \rightarrow -3/2$ , *b*:  $-1/2 \rightarrow -1/2$ , and *c*:  $-1/2 \rightarrow +1/2$ . On the left we show our calculations; on the right we show the experimental data of Shvyd'ko *et al* [13], indicated by dots, together with their calculations, represented by solid traces.

For a thick crystal ( $\beta_{\text{eg}}/2 \gg 1$ ) the integral in (67) is evaluated by the stationary-phase method. When  $0 < \tau \sim 1$  we get (see also [13])

$$j(t) = B + 4A \sin^2\left(\frac{a_{\text{eg}}\tau}{2}\right) J_0^2\left(\sqrt{\beta_{\text{eg}}\tau}\right) \theta(\tau). \quad (70)$$

The flux density  $j(t)$  should still be averaged over  $t$  using some time resolution function of the detector. Following [38], we reduce such averaging to integration:

$$\bar{j}(t) = \frac{1}{\Delta t} \int_{\Delta t} j(t+t') dt' \quad (71)$$

within the time gate of the detector  $\Delta t$ . Numerical calculations of  $\bar{j}(t)$  are compared with experimental data [3] in figure 1 for three exact resonant transitions  $M_g \rightarrow M_e$  ( $a: -1/2 \rightarrow -3/2$ ,  $b: -1/2 \rightarrow -1/2$  and  $c: -1/2 \rightarrow +1/2$ ) in  $^{57}\text{Fe}$  for  $x_0 = 0$ . Here we choose  $\Delta t = 7$  ns and use the parameters  $\beta_a = 32.7$ ,  $\beta_b = 43.6$ ,  $\beta_c = 10.9$ ,  $\Gamma_e = 3.95\Gamma$ ,  $\Gamma = 0.097 \text{ mm s}^{-1}$ ,  $a_a = -105.4$ ,  $a_b = -60.0$ ,  $a_c = -16.0$ , directly measured in [13].

## 5. Conversion electrons

In this section we will analyse the effect of the magnetic field reversal on the yield of conversion electrons during the  $(\gamma, e)$  reaction. In the conversion channel the final states of the whole system will be

$$|b; n\rangle = \psi_{I_g M_g}^N(t) |\{v_s'\}\rangle |0\rangle \phi_f(\mathbf{r}) e^{in\Omega t} \quad (72)$$

where  $|0\rangle$  stands for the vacuum state of the quantized field; the function  $\phi_f(\mathbf{r})$  describes the final state of the emitted conversion electron. Neglecting any influence of the Coulomb and magnetic fields on this function, one has  $\phi_f(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}}$ , where  $\mathbf{q}$  is the wave vector of the emitted electron. Also, the initial wave function (10) should be supplemented by the factor  $\phi_i(\mathbf{r})$  to describe the bound atomic electron.

We consider only the isotope  $^{57}\text{Fe}$  with M1 transitions. Since the electric transitions are not fundamental for this nucleus, the transfer of its energy to the bound electron will be by a two-step process even in the Coulomb gauge. The excited nucleus first emits the transverse photon  $\mathbf{k}'$ ,  $e'_{\lambda'}$ , and this is then absorbed by the electron.

Such  $(\gamma, e)$  reaction is determined by the following  $T$ -matrix:

$$\begin{aligned} \langle\langle b, n | \hat{T} | a, 0 \rangle\rangle &= \sum_{c, m} \sum_{\tilde{c}, \tilde{m}} \langle\langle b, n | \hat{V}_r^c | \tilde{c}, \tilde{m} \rangle\rangle \\ &\times \langle\langle \tilde{c}, \tilde{m} | \hat{G}(\mathcal{E}_a + i\eta) | c, m \rangle\rangle \langle\langle c, m | \hat{V}_r^N | a, 0 \rangle\rangle \end{aligned} \quad (73)$$

where intermediate states of the system are described by the functions

$$\begin{aligned} |c, m\rangle &= \psi_{I_e M_e}^N(t) |\{v_s\}\rangle |0\rangle \phi_i(\mathbf{r}) e^{im\Omega t} \\ |\tilde{c}, \tilde{m}\rangle &= \psi_{I_g M_g}^N(t) |\{v_s''\}\rangle |\mathbf{k}', e'_{\lambda'}\rangle \phi_i(\mathbf{r}) e^{i\tilde{m}\Omega t} \end{aligned} \quad (74)$$

and the quantum numbers  $\mathbf{k}'$ ,  $e'_{\lambda'}$  specify a virtual  $\gamma$ -quantum emitted by the nucleus. Each such function satisfies the eigenvalue equation

$$\tilde{\mathcal{H}}_0 |c, m\rangle = \mathcal{E}_{c, m} |c, m\rangle \quad (75)$$

with quasi-energy  $\mathcal{E}_{c, m} = \mathcal{E}_c + m\hbar\Omega$ .

In the  $T$ -matrix (73), the right-hand matrix element describes absorption of the incident  $\gamma$ -quantum  $\mathbf{k}$ ,  $e$  by the nucleus, while the left-hand one describes absorption of the virtual

photon  $\mathbf{k}'$ ,  $\lambda'$  by an electron. When the system makes a transition from  $|\tilde{c}\rangle$  to  $|b\rangle$ , the state of the nucleus remains the same. Therefore a trivial integration over time gives

$$\langle\langle b, n | \hat{V}_r^e | \tilde{c}, \tilde{m} \rangle\rangle = \langle b | \hat{V}_r^e | \tilde{c} \rangle \delta_{n\tilde{m}}. \quad (76)$$

Calculating the Green matrix, we can use the series presented in chapter 8 of [14]. Then one has

$$\langle\langle b, n | \hat{T} | a, 0 \rangle\rangle = \sum_{c,m} M'_{g'e} \frac{\langle\langle c, m | \hat{V}_r^N | a, 0 \rangle\rangle}{\mathcal{E}_a - \mathcal{E}_{c,m} + i\Gamma/2} \quad (77)$$

where the factor

$$M'_{g'e} = \sum_{\tilde{c}} \frac{\langle b | \hat{V}_r^e | \tilde{c} \rangle \langle\langle \tilde{c}, n | \hat{V}_r^N | c, m \rangle\rangle}{\mathcal{E}_a + i\eta - \mathcal{E}_{\tilde{c},n}} \quad (78)$$

determines the conversion process.

The summation over  $\tilde{c}$  in (78) includes the integration over the wave vectors  $\mathbf{k}'$  of the virtual  $\gamma$ -quanta with energy  $E'$ . Performing such an integration, one can replace the quasi-energies  $\mathcal{E}_{\tilde{c},n}$  by the value  $\mathcal{E}_{\tilde{c}}$ , since  $\hbar\Omega \ll E'$ .

The Raman amplitude of the  $(\gamma, e)$  reaction with emission ( $n > 0$ ) or absorption ( $n < 0$ ) of  $n$  photons with frequency  $\Omega$  by the conversion electron is related to the  $T$ -matrix by

$$f_{gg'}^{(n)}(\mathbf{k}, \mathbf{e}; \mathbf{q}_n) = -\frac{m}{2\pi\hbar^2} \langle\langle b, n | \hat{T} | a, 0 \rangle\rangle \quad (79)$$

where  $q'_n = \sqrt{2m\varepsilon'_n}/\hbar$  is the wave vector of the emitted electron with mass  $m$ . Its kinetic energy

$$\varepsilon'_n = \varepsilon' - n\hbar\Omega \quad (80)$$

where  $\varepsilon'$  is the conversion-electron energy in the absence of reversals, i.e.

$$\begin{aligned} \varepsilon' &= \varepsilon + \gamma_g(M'_g - M_g)h_0 \\ \varepsilon &= \frac{\hbar^2 q^2}{2m} = E - \mathcal{A} \end{aligned} \quad (81)$$

with  $\mathcal{A}$  standing for the binding energy of the electron in the atom. The subscripts of the amplitudes indicate the transition of the nucleus  $M_g \rightarrow M'_g$  during the reaction. It is easily seen that

$$q_n \approx q - n\Omega/v \quad (82)$$

where  $v = \hbar q_0/m$  is the velocity of the electron moving with the energy  $\varepsilon_0 = E'_0 - \mathcal{A}$ .

First we shall consider the electron yield from the atoms of the face layer ( $m = 0$ ). In the slow-collision approximation [39], one has

$$f_{gg'}^{(n)}(\mathbf{k}, \mathbf{e}; \mathbf{q}_n) \sim \sum_{M_g} \sum_{m=-\infty}^{\infty} \frac{a_{eg}^*(m-n)a_{eg}(m)}{E - E'_0 - m\hbar\Omega + i\Gamma/2} M'_{g'e} j^N(\mathbf{k})_{eg}. \quad (83)$$

Here the phonon factors are omitted for brevity. After summation of the cross section over  $\{v'_s\}$  and averaging over  $\{v_s^0\}$ , they give only the Debye–Waller factor  $e^{-2W(\mathbf{k})}$  for the phononless absorption of  $\gamma$ -quanta. The conversion electron emitted by an atom lying on the entrance surface is described by the wave function

$$\psi_{sc}(\mathbf{r}, t)_{M_g \rightarrow M'_g}^{(0)} = \sum_{n=-\infty}^{\infty} f_{gg'}^{(n)}(\mathbf{k}, \mathbf{e}; \mathbf{q}'_n) \frac{1}{r} e^{iq'_n r - i\varepsilon'_n t/\hbar}. \quad (84)$$

When  $T \rightarrow \infty$ , the function  $\psi_{sc}(\mathbf{r}, t)$  can be calculated by analogy with  $E_{sc}(\mathbf{r}, t)$ . However, there is an essential difference in such calculations. Previously we were interested in the

coherent Raman scattering of  $\gamma$ -quanta and therefore have been calculating the coherent waves  $E_{sc}(\mathbf{r}, t)_{coh}^j$ , whose interference ensures the existence of the plane waves inside the crystal. But now we study the conversion process which proceeds only at a single atom. In other words, there is no interference of the electron waves generated by different atoms. Therefore we calculate only the function  $\psi_{sc}(\mathbf{r}, t)_{M_g \rightarrow M'_g}^j$  and do not look for the coherent waves.

The flux density of the conversion electrons emitted from the  $j$ th atom is given by

$$j_{sc}^e(\omega, t)_j = q(2I_g + 1)^{-1} \sum_{M_g, M'_g} \left| \psi_{sc}(\mathbf{r}, t)_{M_g \rightarrow M'_g}^j \right|^2. \quad (85)$$

The corresponding instantaneous differential cross section of the  $(\gamma, e)$  reaction at the  $j$ th atom will be

$$\sigma_{\gamma, e}(\omega, t)^j = \frac{1}{c} j_{sc}^e(\omega, t)_j r^2 \quad (86)$$

where  $c$  is the flux density of the  $\gamma$ -quanta incident on the target. This cross section must be averaged over the energy distribution of incident  $\gamma$ -quanta. As a result one obtains the cross section

$$\bar{\sigma}_{\gamma, e}(t)^j = \int_0^\infty w_e(E) \sigma_{\gamma, e}(\omega, t)^j dE \quad (87)$$

which can be measured experimentally.

Again we assume that  $E \approx E_0^+$  and  $\hbar|\alpha_{eg}| \gg \Gamma$ . Then from (83), (84) one has

$$\begin{aligned} \psi_{sc}(\mathbf{r}, t)_{+M_g \rightarrow +M'_g}^{(0)} &= f_{gg'}(\mathbf{k}, \mathbf{e}; \mathbf{q}') \\ &\times \frac{1}{r} \left\{ (1 - \theta(t - r/v)) e^{iq'r - i\varepsilon't/\hbar} + \theta(t - r/v) e^{iq_0^- r - i\varepsilon_0^- t/\hbar} e^{-\Gamma(t-r/v)/2\hbar} \right\} \end{aligned} \quad (88)$$

and

$$\begin{aligned} \psi_{sc}(\mathbf{r}, t)_{-M_g \rightarrow -M'_g}^{(0)} &= f_{-g, -g'}(\mathbf{k}, \mathbf{e}; \mathbf{q}') \\ &\times \frac{1}{r} \left\{ e^{iq'r - i\varepsilon't/\hbar} - e^{iq_0^+ r - i\varepsilon_0^+ t/\hbar} e^{-\Gamma(t-r/v)/2\hbar} \right\} \theta(t - r/v). \end{aligned} \quad (89)$$

The resonant electron energies take the values

$$\varepsilon_0^\pm = \varepsilon_0 \pm (\gamma_g M'_g - \gamma_e M_e) \hbar_0 \quad (90)$$

where the  $q_0^\pm$  are the corresponding wave vectors. The amplitude  $f_{gg'}(\mathbf{k}, \mathbf{e}; \mathbf{q}')$  describes the  $(\gamma, e)$  reaction in the stationary field  $+\hbar_0$ :

$$f_{gg'}(\mathbf{k}, \mathbf{e}; \mathbf{q}') \sim \sum_{M_e} \frac{M'_g e J_\lambda^N(\mathbf{k})_{eg}}{E - E_0^+ + i\Gamma/2}. \quad (91)$$

From (88), (89) one can see that the electron waves consist of the ordinary wave  $\sim e^{iq'r - i\varepsilon't/\hbar}$  and transients. When the nucleus is in the  $+M_g$ -state, one has at  $t < 0$  the ordinary wave and at  $t > 0$  the transient exponentially attenuating wave packet concentrated at the energy  $\varepsilon_0^-$ . For  $-M_g$ -nuclei, there are both ordinary and transient waves. The latter is concentrated at the energy  $\varepsilon_0^+$ .

The above formulae for  $\psi_{sc}(\mathbf{r}, t)$  are valid only when the incident radiation is described by the function  $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$ . Inside the crystal the electromagnetic wave takes a much more complicated form, being a superposition of plane waves with different frequencies. Calculating  $\psi_{sc}(\mathbf{r}, t)$  in this case, we keep in mind that the transient constituents of  $\mathbf{E}(\mathbf{r}, t)$ ,

appearing after the reversal, generate only ordinary electron waves as if the process takes place in a stationary field. Additionally, we use the relation

$$e^{iqr - i\epsilon t/\hbar} \approx e^{iq_0 r - i\epsilon_0 t/\hbar} e^{-i(\omega - \omega_0)(t - r/v)} \quad (92)$$

resulting from the expansion of  $q$  in terms of  $\epsilon - \epsilon_0$ . Hereafter  $r$  is the distance from the emitting atom.

Then for the conversion-electron wave generated by an atom of the  $m$ th layer of the crystal at  $t > r/v$ , we find the following expressions:

$$\psi_{sc}^{(m)}(\mathbf{r}, t)_{M_g \rightarrow M'_g} = e^{im\mathcal{F}^R} \exp\left(\frac{-i\beta_{eg}(m)/2}{x+i}\right) h(x, \tau) f_{gg}(\mathbf{k}, \mathbf{e}; \mathbf{q}) \frac{1}{r} e^{iq_0^- r - i\epsilon_0^- t/\hbar - \tau'/2} e^{ikmd} \quad (93)$$

and

$$\begin{aligned} \psi_{sc}^{(m)}(\mathbf{r}, t)_{-M_g \rightarrow -M'_g} &= e^{im\mathcal{F}^R} \exp\left(\frac{-i\beta_{eg}(m)/2}{x+i}\right) \\ &\times f_{-g, -g'}(\mathbf{k}, \mathbf{e}; \mathbf{q}) \frac{1}{r} \left\{ e^{iq_0^+ r - i\epsilon_0^+ t/\hbar} - h(x, \tau') e^{iq_0^+ r - i\epsilon_0^+ t/\hbar - \tau'/2} \right\} e^{ikmd} \end{aligned} \quad (94)$$

where

$$h(x, \tau') = 1 - \frac{1}{2} \int_0^1 dy \exp\left(\frac{i\beta_{eg}(m)y/2}{x+i}\right) \sqrt{\frac{\beta_{eg}(m)\tau'}{y}} J_1\left(\sqrt{\beta_{eg}(m)\tau'}y\right) \quad (95)$$

and

$$\tau' = \Gamma(t - r/v)/\hbar.$$

Substituting (93), (94) into (86), (87), one gets the instantaneous averaged cross section for the  $(\gamma, e)$  reaction at an atom of the  $m$ th layer:

$$\begin{aligned} \bar{\sigma}_{\gamma, e}^{(m)}(t) &= \sigma_{res} \exp(-\mu_e md) A \int_{-\infty}^{\infty} \frac{(\gamma_e/\pi) dx}{[(x - x_0)^2 + \gamma_e^2][x^2 + 1]} \exp\left(-\frac{\beta_{eg}(m)}{x^2 + 1}\right) \\ &\times \left\{ |h(x, \tau')|^2 e^{-\tau'} \theta(\tau') + |1 - h(x, \tau') \exp(ix\tau'/2 - \tau'/2) \theta(\tau')|^2 \right\} \end{aligned} \quad (96)$$

where  $\sigma_{res} = \sigma_{\gamma, e}^{(0)}(\omega_0^+, 0)$  stands for the resonant value of the cross section (86) at the entrance surface of the crystal as  $\tau' \leq 0$ .

Figures 2 and 3 illustrate the time dependence of the electron yield from the atoms on the face and exit surfaces, when the resonant transition  $-1/2 \rightarrow -1/2$  is excited in the nucleus at  $t < 0$ . The parameters are the same as for figure 1 for the transition  $b$ .

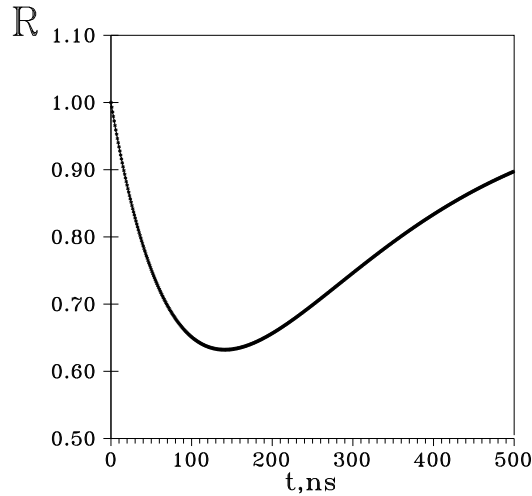
The cross section (96) simplifies for the nuclei at the surface face of the crystal ( $\beta_{eg}(0) = 0$ ). If  $x_0 = 0$ ,  $\gamma_e = 1$ , then

$$\bar{\sigma}_{\gamma, e}^{(0)}(t)/\bar{\sigma}_{\gamma, e}^{(0)}(0) = 1 - \tau' e^{-\tau'} \theta(\tau'). \quad (97)$$

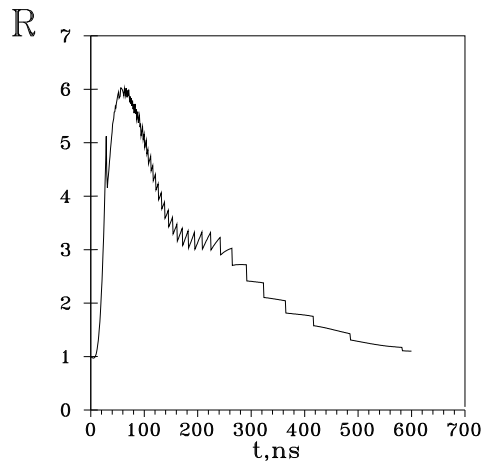
In the opposite limiting case of deep-lying nuclei, when  $\beta_{eg}(m) \gg 1$  and  $0 < \tau' \sim 1$ , we find that

$$\bar{\sigma}_{\gamma, e}^{(m)}(t) \approx 2\sigma_{res} \left(\frac{\tau'}{\beta_{eg}(m)}\right) J_1^2\left(\sqrt{\beta_{eg}(m)\tau'}\right) e^{-\tau'} \theta(\tau'). \quad (98)$$





**Figure 2.** The time dependence of the ratio  $R = \sigma_{\gamma,e}(t)/\sigma_{\gamma,e}(0)$  for the  $(\gamma, e)$  reaction on the entrance surface of the crystal after the reversal of the magnetic field.



**Figure 3.** The time dependence of the ratio  $R = \sigma_{\gamma,e}(t)/\sigma_{\gamma,e}(0)$  for the  $(\gamma, e)$  reaction on the exit surface of the crystal with  $\beta_b = 43.6$  after the reversal of the magnetic field.

## 6. Discussion

Equations (56)–(59) clearly demonstrate that the incident monochromatic electromagnetic wave with frequency  $\omega$  generates inside the crystal the well-known ordinary wave  $\sim e^{-i\omega t}$  and two transient wave packets with carrier frequencies  $\omega_0^\pm$ , arising after the reversal of the magnetic field. The waves  $\mathbf{E}_{tr}(\tilde{t})^{(+)}$  are emitted by the nuclei excited prior to the reversal via the transition  $+M_g \rightarrow M_e$ . Their de-excitation to the same state  $M_g$  occurs with the resonant frequency  $\omega_0^-$  corresponding to new value of the field  $-\mathbf{h}_0$ . The wave  $\mathbf{E}_{tr}(\tilde{t})^{(-)}$  results from the sudden opening of the transition at  $t = 0$  for those nuclei which are initially in the state  $-M_g$  and do not interact with incident radiation at  $t < 0$ . The interference of all of these waves provides a flash of the beam intensity transmitted through a thick crystal

just after the reversal of the field. Such a flash is followed by attenuating oscillations of the intensity. This phenomenon, discovered in [12, 13], is illustrated by figure 1, where one can see good agreement of the calculations with experiment [13]. Note that we used only one fitting parameter  $\Delta t$ , while in [13] all of the parameters  $a_{eg}$  and  $\beta_{eg}$  were slightly varied. The coherent wave  $E(z, t)_{coh}$  is a result of averaging over the magnetic quantum numbers  $M_g$  of the waves  $E(z, t)_{M_g \rightarrow M_g}$ . Therefore it is a sum of all three waves  $E_{ord}$  and  $E_{tr}^{(\pm)}$ .

But there is no coherence of the electron waves emitted by different atoms. Therefore we average over  $M_g$  just the cross section for this process and not the waves. As a consequence, the cross section is proportional to the sum of electron fluxes emitted by the atoms with nuclei in the initial states  $+M_g$  and  $-M_g$ . The flux from the atom with the nucleus  $-M_g$  at  $t > 0$  is formed by two waves (ordinary and transient), whose interference provides some inhibition of the electron yield (see figure 2) within a time interval of the order of the nuclear lifetime. Such suppression should take place only for electrons being emitted from atoms lying near the surface face of the crystal.

All of the calculations given above for the conversion process are valid for any other inelastic process—say, for the inelastic scattering of  $\gamma$ -quanta when the nucleus changes its orientation ( $M_g \rightarrow M'_g \neq M_g$ ). The suppression of inelastic channels and reactions caused by the magnetic field reversal is a new effect, unlike the well-known suppression effect occurring in the diffraction of  $\gamma$ -quanta or neutrons in a perfect crystal [40–44]. The latter is analogous to the Borrmann effect [45] of the anomalous transmission of x-rays through a crystal in the case of Laue diffraction, which is explained by the formation of two standing waves inside the crystal. A similar effect also arises in inelastic two-wave diffraction [22, 23]. In contrast, the suppression effect predicted above is due to the interference of the ordinary and transient waves. This may be observed far from the Bragg condition and even in imperfect crystals or in amorphous magnetic films.

At the same time, figure 3 shows an increase of the conversion-electron yield from the atoms lying at the back surface of the crystal after the jump of the magnetic field. This is ensured by a great flash of the phononless component of the coherent Mössbauer radiation intensity deep inside the crystal, where the amplitudes of the transient and ordinary electromagnetic waves become comparable. This great enhancement prevails over the small suppression of the electron yield.

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